Trigonometry Word Problems

1. Juan and Romella are standing at the seashore 10 miles apart. The coastline is a straight line between them. Both can see the same ship in the water. The angle between the coastline and the line between the ship and Juan is 35 degrees. The angle between the coastline and the line between the ship and Romella is 45 degrees. How far is the ship from Juan?

\[
\begin{align*}
35^\circ + 45^\circ + \theta &= 180^\circ \\
60^\circ + \theta &= 180^\circ \\
\theta &= 100^\circ
\end{align*}
\]

\[
\frac{\sin 45^\circ}{\sin 100^\circ} \times 10 = \frac{10 \sin 45^\circ}{\sin 100^\circ}
\]

\[
\theta = 10^\circ
\]

2. A triangular playground has sides of lengths 475 feet, 595 feet, and 401 feet. What are the measures of the angles between the sides, to the nearest tenth of a degree?

\[
475^2 = 401^2 + 595^2 - 2(401)(595) \cos \alpha
\]

\[
\frac{475^2 - 401^2 - 595^2}{-2(401)(595)} = \cos \alpha
\]

\[
\alpha \approx 57^\circ
\]

\[
\frac{\sin B}{595} = \frac{\sin 52^\circ}{475}
\]

\[
\angle B \approx 80^\circ
\]

\[
52^\circ + 80^\circ + \angle C = 180^\circ
\]

\[
\angle C \approx 48^\circ
\]

3. Polly wants to know the area of the triangular sail on her sailboat because she is buying a new sail. Two sides are 15 feet and 18 feet and the angle between them is 75°. What is the area of the sail?

\[
A = \frac{1}{2} (15)(18) \sin 75^\circ
\]

\[
A \approx 130.4 \text{ ft}^2
\]

4. Matt measures the angle of elevation of the peak of a mountain as 35°. Susie, who is 1200 feet closer on a straight level path, measures the angle of elevation as 42°. How high is the mountain?

\[
\frac{\sin 7^\circ}{1200} = \frac{\sin 35^\circ}{Y}
\]

\[
\frac{Y \sin 7^\circ}{\sin 7^\circ} = \frac{1200 \sin 35^\circ}{\sin 7^\circ}
\]

\[
Y \approx 5,648
\]

\[
\sin 42^\circ = \frac{x}{5,648}
\]

\[
x = 5,648 \sin 42^\circ
\]

\[
\approx 3,779 \text{ ft}
\]
5. A rock climber is part of the way up a climb when he can see both the peak and the base of the Gray Mountain. When viewing the peak of the mountain, his angle of elevation is 42°. When viewing the base of the mountain, his angle of depression is 36°. If he knows the Gray Mountain is 2000 feet high and the base of the mountain is at sea level, then what is the elevation of the climber to the nearest foot?

\[
\begin{align*}
\angle C &= 42° + 36° = 78° \\
\frac{\sin 78°}{2000} &= \frac{\sin 48°}{y} \\
\frac{y \sin 78°}{2000} &= \frac{\sin 48°}{\sin 78°} \\
y &= 1.519.5 \text{ ft}
\end{align*}
\]

\[\cos 54° = \frac{x}{1.519.5}, \quad x = 893 \text{ ft}\]

6. Two radar stations 2.4 miles apart are tracking an airplane. The straight-line distance between Station A and the plane is 7.4 miles. The straight-line distance between Station B and the plane is 6.9 miles. What is the angle of elevation from Station A to the plane? Round to the nearest degree.

\[a^2 = b^2 + c^2 - 2bc \cos A\]

\[6.9^2 = 2.4^2 + 7.4^2 - 2(2.4)(7.4) \cos \theta\]

\[
\frac{6.9^2 - 2.4^2 - 7.4^2}{-2(2.4)(7.4)} = \cos \theta
\]

\[\theta \approx 69°\]

7. During a figure skating routine, Jackie and Peter skate apart with an angle of 15° between them. Jackie skates for 5 meters and Peter skates for 7 meters. How far apart are the skaters?

\[a^2 = 7^2 + 5^2 - 2(7)(5) \cos 15°\]

\[= 49 + 25 - 70 \cos 15°\]

\[a = 16.4\]

\[a = 2.5 \text{ m}\]